$$\sum_{n=1}^{40000} \frac{1}{\sqrt{n}}$$ の整数部分を求めよ

$$S = \sum_{n=1}^{40000} \frac{1}{\sqrt{n}} = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \left(\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} + \cdots + \frac{1}{\sqrt{40000}}\right)$$

$= 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{3} + \frac{1}{2} + S_5$ とします。 --- ①

$$S_5 = \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} + \cdots + \frac{1}{\sqrt{40000}}$$

さて $\int_{n}^{n+1} \frac{dx}{\sqrt{x}} < \frac{1}{\sqrt{n}} < \int_{n-1}^{n} \frac{dx}{\sqrt{x}}$ から

$$S_5 > \int_5^6 \frac{dx}{\sqrt{x}} + \int_6^7 \frac{dx}{\sqrt{x}} + \cdots + \int_{40000}^{40001} \frac{dx}{\sqrt{x}} = \int_5^{40001} \frac{dx}{\sqrt{x}} = 2\sqrt{40001} - 2\sqrt{5} \quad ②$$

$$S_5 < \int_4^5 + \int_5^6 + \cdots + \int_{39999}^{40000} \frac{dx}{\sqrt{x}} = \int_4^{40000} \frac{dx}{\sqrt{x}} = 2\sqrt{40000} - 2\sqrt{4} = 396 \quad ③$$

$S_5 > 2\sqrt{40001} - 2\sqrt{5} > 2\times\sqrt{40000} - 2\times 2.2361 = 400 - 4.4722$

$\sqrt{5} \fallingdotseq 2.2360679$

$S_5 > 395.5278$ ④

③, ④ から $\therefore\ 395.5278 < S_5 < 396$ --- ⑤

ところで $1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{3} + \frac{1}{2} \fallingdotseq 1.5 + \frac{1.4142}{2} + \frac{1.7321}{3}$

$= 1.5 + 1.284 \fallingdotseq 2.784$ ⑥

$$\begin{array}{r} 395.5278 \\ +\ \ 2.784 \\ \hline 398.3118 \end{array} \qquad \begin{array}{r} 396 \\ +\ 2.784 \\ \hline 398.784 \end{array}$$ から

$$398.3 < S < 398.8$$

となって <u>Sの整数部分は 398</u>